Pair Annihilation Effects on Lower Hybrid Oscillation in Semi-Bounded Magnetized Dusty Pair Plasmas

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Z. Naturforsch. **61a,** 667 – 671 (2006); received October 10, 2006

The electron-positron pair annihilation effects on the electrostatic hybrid resonance oscillation are investigated in semi-bounded magnetized dusty pair plasmas. The surface wave dispersion relation is obtained by the plasma dielectric function with the specular reflection condition. The result shows the existence lower hybrid resonance oscillation modes in semi-bounded dusty pair plasmas. It is found that the electron-positron annihilation events enhance the lower hybrid resonance oscillation frequency. It is also found that the lower hybrid resonance frequency decreases with increasing the ratio of the positron density to the electron density. In addition, the lower hybrid resonance frequency decreases with increasing the strength of the magnetic field.

Key words: Surface Waves; Pair-Ion-Dust Plasmas.

1. Introduction

The surface plasma waves [1 – 3] in bounded and semi-bounded plasmas have been of great interest since the frequency spectra of the surface waves provide useful information on various plasma parameters. Recently, there has been a considerable interest in the dynamics of plasmas containing charged dust grains including collective effects and strong electrostatic interaction between the dust grains. These dust grains are ubiquitous in the universe and are known to play important roles in astrophysical plasmas [4 – 6]. It is known that the electron-positron-ion-dust plasmas have been encountered in astrophysical environments such as active galactic nuclei, atmosphere of neutron stars, pulsar magnetospheres, and supernova environments as well as in laboratory experiments such as semiconductor plasmas and cluster explosions by intense laser beams [7, 8]. The detailed charge distributions in the pulsar magnetic magnetosphere by the Goldreich-Julian model is well known [9, 10]. Recently, the surface wave propagations have been extensively investigated in dusty pair plasmas [11, 12]. In addition, the hybrid resonance oscillation modes have been found in both astrophysical and laboratory plasmas [13]. The direct and indirect annihilations of positrons with electrons are now of great interests and have been investigated in astrophysical plasmas [14 – 17]. However, to the best of our knowledge, the electron-positron pair annihilation effects on the electrostatic hybrid resonance oscillation in semi-bounded magnetized pair-ion-dust plasmas have not been investigated yet. It is quite obvious that the theoretical investigation on the dispersion properties of the surface wave in bounded plasmas can be a useful tool for investigating the structure and physical properties of such plasmas. Thus in this paper, we investigate the electron-positron pair annihilation effects on the lower hybrid resonance oscillation in semi-bounded magnetized dusty pair plasmas using the specular reflection condition with the plasma dielectric function.

In Section 2, we discuss the lower hybrid resonance oscillation in a semi-bounded magnetized dusty pair plasma using the specular reflection boundary condition and the longitudinal plasma dielectric function. In Section 3, we obtain the dispersion relation of the lower hybrid resonance oscillation. We also investigate the variation of the resonance oscillation mode due to the pair annihilation effects. Finally, the summary and conclusion are given in Section 4.

2. Spectral Reflection Condition

It is known that the specular reflection condition [1, 2] is quite useful to investigate the dispersion relation for various electromagnetic and electrostatic surface waves in bounded and semi-bounded plasmas. The dispersion relation for surface waves propagating in the z-direction in semi-bounded isotropic plasmas with the plasma-vacuum interface at $x = 0$ is given by the specular reflection condition [1] which can be used for the boundary condition on a small perturbation distribution:

$$\frac{\omega}{\pi c}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_x}{k^2}\left[\frac{k_z^2c^2}{\omega^2\varepsilon_\mathrm{l}(\omega,k)}-\frac{k_x^2c^2}{k^2c^2-\omega^2\varepsilon_\mathrm{t}(\omega,k)}\right]\left(\frac{k_z^2c^2}{\omega^2}-1\right)^{1/2}=0, \tag{1}$$

where $\varepsilon_\mathrm{l}(\omega,k)$ and $\varepsilon_\mathrm{t}(\omega,k)$ are the longitudinal and transverse components of the plasma dielectric function, ω is the frequency, c the speed of the light, and k [$=(k_x^2+k_z^2)^{1/2}$] the wave number. In this situation, the y-coordinate is a translational invariance and can be ignored without loss of generality. It is known that the physical properties of electrostatic waves in plasmas would be determined by the plasma dielectric function. When the external magnetic field is parallel to the surface boundary, the plasma dielectric function in magnetized dusty pair plasmas is given by

$$\varepsilon_\mathrm{l}(\omega,k_\perp,k_\parallel)=1+\chi_++\chi_-+\chi_\mathrm{i}+\chi_\mathrm{d}. \tag{2}$$

Here, χ_s ($\mathrm{s}=\mathrm{e}^+,\mathrm{e}^-,\mathrm{i},\mathrm{d}$) are the plasma dielectric susceptibilities [18] for positrons (+), electrons (−), ions (i), and dust grains (d):

$$\chi_\mathrm{s}(\omega,k_\perp,k_\parallel)=\frac{\omega_\mathrm{ps}^2}{k^2v_\mathrm{Ts}^2}\left[1-\omega\sum_{n=-\infty}^{\infty}I_n(k_\perp^2v_\mathrm{Ts}^2/\omega_\mathrm{cs}^2)\exp(-k_\perp^2v_\mathrm{Ts}^2/\omega_\mathrm{cs}^2)\int_{-\infty}^{\infty}\frac{\mathrm{d}v_z f_s(v_z)}{\omega-k_\parallel v_z-n\omega_\mathrm{cs}}\right], \tag{3}$$

where ω_ps is the plasma frequency of the species s, v_Ts the thermal velocity, I_n the modified Bessel function of order n, ω_cs the gyrofrequency, and $f_\mathrm{s}(v_z)$ [$=(2\pi v_\mathrm{Ts}^2)^{-1/2}\exp(-v_z^2/2v_\mathrm{Ts}^2)$] the normalized Maxwellian velocity distribution function, $k_\perp=k_x$, and $k_\parallel=k_z$. For kv_Ts, ω_cd, $\omega_\mathrm{ci}\ll\omega\ll\omega_\mathrm{ce}$, the plasma dielectric susceptibilities for the electrons and positrons become $\chi_\mathrm{s}\approx\omega_\mathrm{ps}^2k_\perp^2/\omega_\mathrm{cs}^2k^2-\omega_\mathrm{ps}^2k_\parallel^2/\omega^2k^2$, $\chi_\mathrm{i}\approx-\omega_\mathrm{pi}^2/\omega^2$, and $\chi_\mathrm{d}\approx-\omega_\mathrm{pd}^2/\omega^2$ [19]. After some algebraic manipulations, the plasma dielectric function for the lower hybrid resonance oscillation in dusty pair plasmas is found to be

$$\begin{aligned}\varepsilon_\mathrm{l}(\omega,k_\perp,k_\parallel,\xi)=1&+\frac{\omega_\mathrm{pe^+}^2k_\perp^2}{\omega_\mathrm{ce^+}^2k^2}(1-\xi n_-/n_+)-\frac{\omega_\mathrm{pe^+}^2k_\parallel^2}{\omega^2k^2}(1-\xi n_-/n_+)\\&+\frac{\omega_\mathrm{pe^-}^2k_\perp^2}{\omega_\mathrm{ce^-}^2k^2}(1-\xi)-\frac{\omega_\mathrm{pe^-}^2k_\parallel^2}{\omega^2k^2}(1-\xi)-\frac{\omega_\mathrm{pi}^2}{\omega^2}-\frac{\omega_\mathrm{pd}^2}{\omega^2},\end{aligned} \tag{4}$$

where n_- and n_+ are the electron and positron densities, respectively, $\xi\equiv\Delta n/n$, $\Delta n\equiv\Delta n_-=\Delta n_+$, Δn_- and Δn_+ are the density reductions of electrons and positrons due to the pair annihilations. In this work, we only consider the direct annihilations of positrons with free electrons, and the single photon annihilations of positrons with bound electrons have been neglected since the single photon positron annihilation cross-section with a bound electron is quite small compared with the two-photon positron annihilation cross-section with a free electron [16, 20]. The detailed discussion on the general annihilation cross-section including the Coulomb focusing and the relationship to the pair annihilation and production can be found in a recent excellent book by Gould [17].

3. Dispersion Relation

In the quasi-static limit ($\omega^2\varepsilon/c^2\ll k^2$), the dispersion equation for the electrostatic lower hybrid resonance oscillation in semi-bounded magnetized pair-ion-dust plasmas is given by the contour integration

$$\oint \mathrm{d}k_\perp k_\| \left\{ \left[\left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) \right) - \left(\frac{\omega_{\mathrm{pi}}^2}{\omega^2} + \frac{\omega_{\mathrm{pd}}^2}{\omega^2} \right) \right] k_\perp^2 + \left[1 - \left(\frac{\omega_{\mathrm{pe}}^2}{\omega^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) + \frac{\omega_{\mathrm{pi}}^2}{\omega^2} + \frac{\omega_{\mathrm{pd}}^2}{\omega^2} \right) \right] k_\|^2 \right\}^{-1} + \pi = 0, \tag{5}$$

where $\omega_{\mathrm{pe}} \equiv \omega_{\mathrm{pe}^+}$ and $\omega_{\mathrm{ce}} \equiv \omega_{\mathrm{ce}^+}$. By using the contour integration in the complex $k_\perp$-plane with the Jordan's lemma [21], the dispersion relation is found to be

$$\begin{aligned} &\left(\frac{\omega}{\omega_{\mathrm{pi}}} \right)^4 \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) \\ &- \left(\frac{\omega}{\omega_{\mathrm{pi}}} \right)^2 \left[\left(1 + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) + \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) \right) \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{pi}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) \right] \\ &+ \left(1 + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{pi}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) = 0. \end{aligned} \tag{6}$$

From the dispersion relation, the frequency for the lower hybrid resonance oscillation mode becomes

$$\begin{aligned} \frac{\omega(n_+/n_-, \xi)}{\omega_{\mathrm{pi}}} = &\left\{ \left(1 + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) + \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) \right) \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{pi}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) \right. \\ &- \left[\left[\left(1 + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) + \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) \right) \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{pi}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) \right]^2 \right. \\ &\left. \left. - 4 \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) \left(1 + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) \left(1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{pi}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) + \frac{\omega_{\mathrm{pd}}^2}{\omega_{\mathrm{pi}}^2} \right) \right]^{1/2} \right\}^{1/2} \\ &\cdot \left[2 \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2} \left(1 + \frac{n_+}{n_-} - 2\xi \right) \right]^{-1/2}. \end{aligned} \tag{7}$$

Figure 1 shows the three-dimensional plot of the lower hybrid resonance frequency as a function of $\omega_{\mathrm{pe}}/\omega_{\mathrm{ce}}$ and $\Delta n/n_-$. Figure 2 represents the lower hybrid resonance frequency as a function of n_+/n_- for various values of $\Delta n/n_-$. As we see in theses figures, the electron-positron pair annihilation events enhance the lower hybrid resonance frequency since the resonance frequency increases with increasing the density reduction Δn. Figure 3 shows the three-dimensional plot of the lower hybrid resonance frequency as a function of n_+/n_- and $\omega_{\mathrm{pe}}/\omega_{\mathrm{ce}}$. It is found that the lower hybrid resonance frequency decreases with increasing the ratio of the positron density to the electron density (n_+/n_-). In addition, the lower hybrid resonance frequency decreases with increasing $\omega_{\mathrm{pe}}/\omega_{\mathrm{ce}}$, i. e., the strength of the external magnetic field. Figure 4 shows the three-dimensional plot of the lower hybrid resonance frequency as a function of $\Delta n/n_-$ and $\omega_{\mathrm{pi}}/\omega_{\mathrm{pe}}$. It is also found that the resonance frequency is almost independent of the frequency ratio $\omega_{\mathrm{pi}}/\omega_{\mathrm{pe}}$. In this work we only consider the variation of the lower hybrid oscillation frequency due to the electron-positron annihilation events. However, monitoring the spectral variation from the astrophysical compact objects, the actual time evolution of the lower hybrid oscillation frequency should be considered in order to determine the particle compositions and the rate of annihilation. Therefore, in future work, the time variation of the lower hybrid oscillation frequency, i. e., $\mathrm{d}\omega(n_+/n_-, \xi)/\mathrm{d}t$, would be considered in elsewhere including the direct electron-positron annihilation, the formation of the positronium, and the positron annihilation with the bound atomic electrons in partially ionized plasmas.

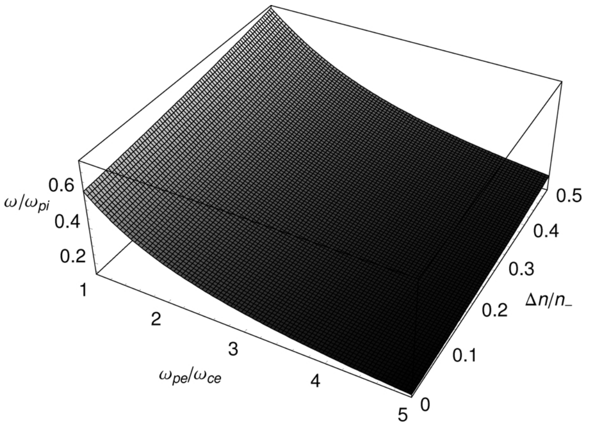

Fig. 1. The three-dimensional plot of the lower hybrid resonance frequency as a function of $\omega_{\rm pe}/\omega_{\rm ce}$ and $\Delta n/n_-$ when $n_+/n_- = 0.8$, $\omega_{\rm pd}/\omega_{\rm pi} = 10^{-3}$, and $\omega_{\rm pi}/\omega_{\rm pe} = 0.05$.

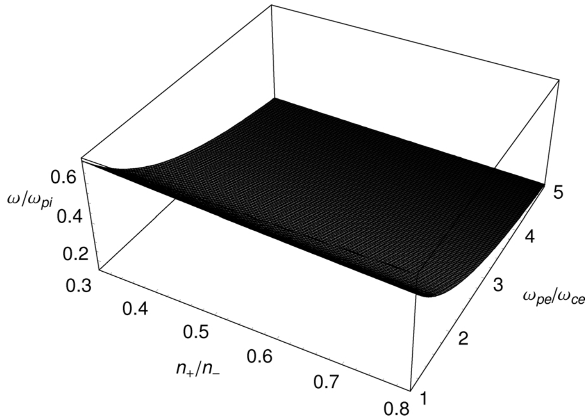

Fig. 3. The three-dimensional plot of the lower hybrid resonance frequency as a function of n_+/n_- and $\omega_{\rm pe}/\omega_{\rm ce}$ when $\Delta n/n_- = 0.15$, $\omega_{\rm pd}/\omega_{\rm pi} = 10^{-3}$, and $\omega_{\rm pi}/\omega_{\rm pe} = 0.05$.

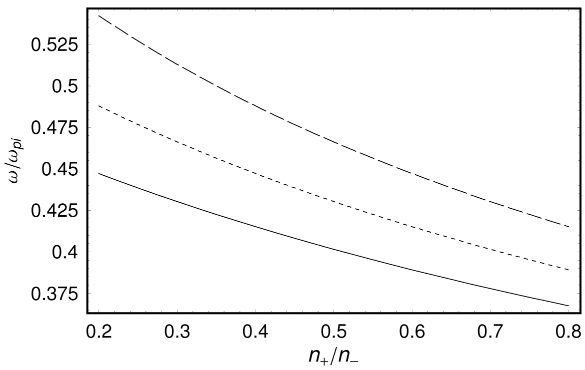

Fig. 2. The lower hybrid resonance frequency as a function of n_+/n_- for $\omega_{\rm pe}/\omega_{\rm ce} = 2$, $\omega_{\rm pd}/\omega_{\rm pi} = 10^{-3}$, and $\omega_{\rm pi}/\omega_{\rm pe} = 0.05$. The solid line represents the case of $\Delta n/n_- = 0.1$. The dotted line represents the case of $\Delta n/n_- = 0.2$. The dashed line represents the case of $\Delta n/n_- = 0.3$.

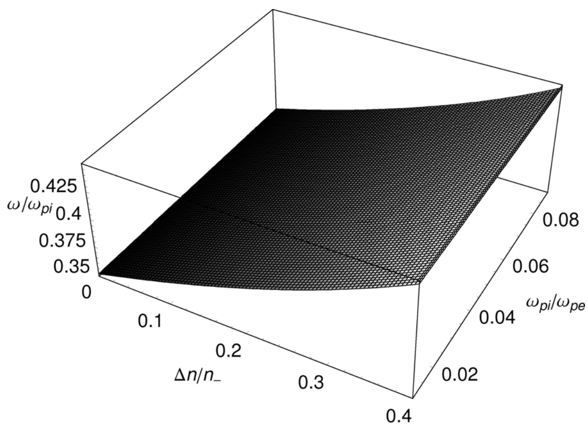

Fig. 4. The three-dimensional plot of the lower hybrid resonance frequency as a function of $\Delta n/n_-$ and $\omega_{\rm pi}/\omega_{\rm pe}$ when $n_+/n_- = 0.8$, $\omega_{\rm pe}/\omega_{\rm ce} = 2$, and $\omega_{\rm pd}/\omega_{\rm pi} = 10^{-3}$.

4. Summary and Conclusion

We investigate the electron-positron pair annihilation effects on the electrostatic hybrid resonance oscillation in semi-bounded magnetized dusty pair plasmas. The dispersion relation of the electrostatic hybrid resonance oscillation is obtained by the plasma dielectric function with the specular reflection boundary condition. This result shows the existence lower hybrid resonance oscillation modes in semi-bounded dusty pair plasmas. It is found that the electron-positron annihilation events enhance the lower hybrid resonance oscillation frequency. It is interesting to note that the lower hybrid resonance frequency decreases with increasing the ratio of the positron density to the electron density. In addition, the lower hybrid resonance frequency decreases with increasing $\omega_{\rm pe}/\omega_{\rm ce}$, i. e., the strength of the external magnetic field. It is also found that the resonance frequency is almost independent of the frequency ratio $\omega_{\rm pi}/\omega_{\rm pe}$. Hence, it is found that the electron-positron pair annihilation plays a significant role in dispersion properties of the surface wave in bounded electron-positron pair plasmas. In the future, we may examine and resolve the spectrum due to the interaction between the surface wave and the gamma-ray emission [22] caused by the photons created by the pair annihilation in electron-positron plasmas. These results provide useful information on the pair annihilation effects of the electrostatic hybrid resonance oscillation in semi-bounded magnetized dusty pair plasmas.

Acknowledgements

The authors gratefully acknowledge Professor W. Hong for useful discussions on the surface waves. One of the authors (Y.-D. J.) also thanks Professor H. J. Lee for useful discussions on dispersion relations in bounded plasmas. He also thanks Dr. S. L. O'Dell for useful discussions on the pulsar magnetosphere and for warm hospitality while visiting NASA/Marshall Space Flight Center. The authors would like to thank the anonymous referee for suggesting improvements to this text.

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) (KRF-2006-311-C00031) and by the Korean Ministry of Education through the 2nd stage of the Brain Korea 21 Project in 2006.

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